Mansoura Engineering Journal (MEJ) Vol. 19, No. 2, June 1994 E. 23

DYNAMIC PERFORMANCE ANALYSIS OF INVERTER-FED PERMANENT MAGNET SYNCHRONOUS MOTOR

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تحليل الأداء الديناميكى لمحرك تزامنى ذو أقطاب دائمة يغذى من عاكس الكترونى

الخلاصة:
يقدم هذا البحث تحليل نظرى للأداء الديناميكى لمحرك تزامنى ذو أقطاب دائمة يغذى من عاكس الكترونى مستخدما أسلوب العزوم المتزامنة والمخمدة. طبقت طريقة عددية لاستنتاج أداء المحرك التزامنى ذو الأقطاب الدائمة. تم الحصول على مجموعة شاملة لنتائج التمثيل الرقمى للاداء الديناميكى للمحرك مع إستخدام أو بدون العاكس الألكترونى. أمكن إيجاد وعرض الأشكال الموجية للجهد ، التيارات المباشرة والمتعامدة والعزوم عند حالات تشغيل مختلفة ومقارنتها لتوضيح الاختلافات الناتجة عند إستخدام العاكس الألكترونى. نوقشت أيضا تأثير الترددات نتيجة إستخدام العاكس الألكترونى على الأداء الديناميكى للمحرك وكذلك إيجاد المدى الأمثل للتردد.

ABSTRACT

This paper analyze the dynamic performance of inverter fed permanent magnet synchronous motors (PMSM) using damping and synchronizing torques. A numerical algorithm has been applied to predict the performance using overall nonlinear model of permanent magnet synchronous motor. A comprehensive set of results for the digital simulation of the steady state and dynamic performances for a motor with and without pulse-width modulation inverter is obtained. The voltage, direct and quadrature current, and torque waveforms at different modes of operations are presented in a comparative form, illustrating the differences caused by introducing the inverter. The effects of frequency variation due to the use of inverter on motor dynamic performance, and the optimum range of frequency is also discussed.

1. INTRODUCTION

Progress in the development of electrical machines employing permanent magnet PM excitation has rapidly increased in recent years. The main responsible factor is the tremendous improvement in the permanent magnet technology and availability of new PM materials. Other contributory factors include improved efficiency, increased reliability and reduced manufacturing costs.

Modeling of permanent magnet synchronous motors PMSM, and their steady state performance have been extensively examined and useful results have been reported [1-4]. Problems associated with the starting and dynamic performance have been also discussed in detail [5-8]. The effects of varying supply frequency on motor parameters have been studied by Chalmers [9]. The steady state characteristics of PMSM when operated from a 180° inverter with phase controller has been considered by Krause et.al [10]. A speed control using the boundary layer state observer is presented by Cho et.al [11]. For a high performance drive, appropriate choice of the power converter and controller using computer simulation of the drive system is achieved by Dhaouadi et.al [12]. However, these previous published works have not investigated the dynamic performance analysis of the pulse-width modulation(PWM) inverter-fed PMSM when the motor is subjected to a small disturbance in speed or following a small load change.

The purpose of this paper is to analyze the steady state and dynamic performance of the (PWM)inverted fed PMSM using damping and synchronizing torques technique. A comparison of steady state and dynamic performance when the motor is operated with and without inverter is presented. The effects of frequency variation on the motor variables and response are illustrated and the optimum range of operating frequencies are defined.

2. NONLINEAR MOTOR EQUATIONS

The generalized d-and q-axes nonlinear differential equations that describe the performance of PMSM are given for a fixed rotor reference frame in per unit form. Details of the transformations used as well as assumptions made are given in [13]. The final set of equations are :-

$$V_d = -V \sin \delta = R i_d + p\psi_d + \psi_q \, p\theta \tag{1}$$

$$V_q = V \cos \delta = R i_q + p\psi_q + \psi_d \, p\theta \tag{2}$$

$$0 = R_D i_D + p\,\psi_D \tag{3}$$

$$0 = R_Q i_Q + p\,\psi_Q \tag{4}$$

with a fixed magnet strength excitation represented by an equivalent field current i_f. The axes fluxes can be given as following :

$$\psi_d = X_d \, i_d + X_{ad} \, i_D + X_{ad} \, i_f \tag{5}$$

$$\psi_D = X_D \, i_D + X_{ad} \, i_d + X_{ad} \, i_f \tag{6}$$

$$\psi_q = X_q \, i_q + X_{aq} \, i_Q \tag{7}$$

$$\psi_Q = X_Q \, i_Q + X_{aq} \, i_q \tag{8}$$

The mechanical equations can also be written as

$$T_M = \psi_d \, i_q - \psi_q \, i_d \tag{9}$$

$$p\,\omega_M = (T_M - T_L)/ J \tag{10}$$

$$\omega_M = p\,\theta \tag{11}$$

A digital computer program has been developed to solve the above nonlinear equations, using a standard numerical integration technique based on Runge - Kutta algorithm.

3. DAMPING AND SYNCHRONIZING TORQUE TECHNIQUE

The mechanical equation in a PMSM, following a small disturbance, can be written as follows :

$$\Delta \dot{\omega}_M = (\Delta T_M - \Delta T_L) / J \tag{12}$$

The change in the electrical torque, at any frequency of rotor motion, can be decomposed into two components, a synchronizing torque component in phase with the rotor angle and a damping torque component in phase with the rotor speed. Modeling of damping and synchronizing torques and their accurate prediction provides physical realization quantitative assessment of dynamical performance under different operating conditions. This section is concerned with the development of a numerical algorithm to model both damping and synchronizing torques based on a time domain analysis of the nonlinear response. The above torque components can be represented by the synchronizing torque coefficient K_s and the damping torque coefficient K_D [8]

$$\Delta T_M \ (t) = K_D \ . \ \Delta\omega(t) + K_S \ . \ \Delta\delta \ (t) \tag{13}$$

Where K_D and K_s are damping and synchronizing torque coefficients which must be positive for a stable motor.
For the purpose of computing these coefficients the error between the actual torque deviation and that obtained by summing both the damping and synchronizing components can be estimated. Minimisation of the square of the errors, over a period of t, leads to the following equations [8] :

$$\sum^{N} \Delta \, T_M(t).\Delta\delta(t) = K_s \sum^{N} [\Delta\delta(t)]^2 + K_D \sum^{N} \Delta\omega(t).\Delta\delta(t) \tag{14}$$

$$\sum^{N} \Delta \, T_M(t).\Delta\omega(t) = K_D \sum^{N} [\Delta\omega(t)]^2 + K_s \sum^{N} \Delta\omega(t).\Delta\delta(t) \tag{15}$$

Note that t= N.T, where N is the number of iteration, and T is the integration step. Solving the above two equations (14 & 15) gives the time invariant values of torque coefficients K_D and K_s.

4. VOLTAGE - FREQUENCY CONTROL

4.1 System Description

The permanent magnet synchronous motor is fed via a 3-phase fully controlled bridge rectifier, d.c link and pulse-width modulation inverter as shown in Fig. (1). The speed of the motor is proportional to the inverter frequency. The mean d.c voltage is controllable by delaying the commutation of the thyristors by the firing delay angle α. The output d.c voltage of controlled rectifier bridge is given by :

$$E_{d.c} = 3 \frac{\sqrt{6}}{\pi} V \cos \alpha \tag{16}$$

Inverters convert d.c power to a.c power at desired output voltage and frequency. In most inverter applications it is necessary to be able to control both the output voltage and the output frequency to keep (V/f) = constant. The uncontrollable voltage requirement may arise out of the need to maintain constant flux in a.c motors driven at variable frequency. If the d.c input voltage is controllable, then an inverter with a fixed ratio of the d.c input voltage to a.c output voltage may be satisfactory. If the d.c input voltage is not controllable, then control of the output voltage and output frequency must be obtained by employing pulse-width modulation inverter.

4.2 Symmetrical Pulse - Width Voltage Control

The pulse - width modulation technique is presently the most popular and economical method of voltage and frequency control. The technique of symmetrical multiple pulse -width voltage control can easily be applied to the three phase circuits of the half-bridge and full bridge three phase inverters. In the half- bridge circuits the line to line voltage is limited to a quasi-rectangular waveform as the maximum conduction interval for each half-cycle is 120°. To vary the voltage from this maximum value, the output voltage waveform is divided into a number of symmetrical pulses whose widths are varied to achieve the desired voltage control [13].

To obtain symmetrical pulses in the three phase output of the half-bridge inverter, the modulating frequency is constructed to be 6 mf, where m = 1, 2, 3. ... and f is the desired output frequency. The number of pulses per half-cycle of the line to line voltage is given by 2 m. The output voltage is controlled by varying the width of the pulses symmetrically. In some applications the number of pulses per half-cycle is kept fixed at all voltages. More complex schemes allow for increasing the number of pulses per half-cycle at lower output voltages to reduce the harmonic content of the waveform. However, the relative pulse-width is defined as:

$$rpw = \beta / \tau \tag{17}$$

Where

τ : is the theoretical maximum pulse - width ($\tau=\pi/3m$)

β : is the variable pulse - width.

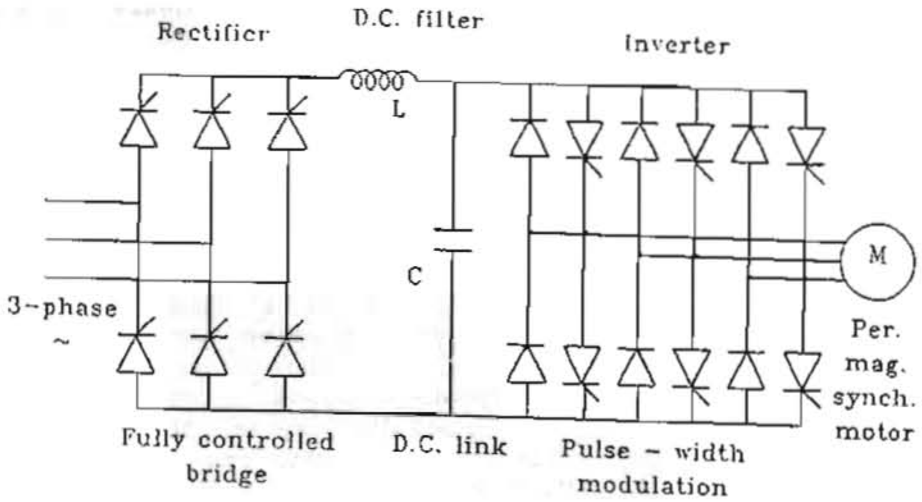

Fig. (1) Variable-voltage variable-frequency control of permenant magnet synchronous motor

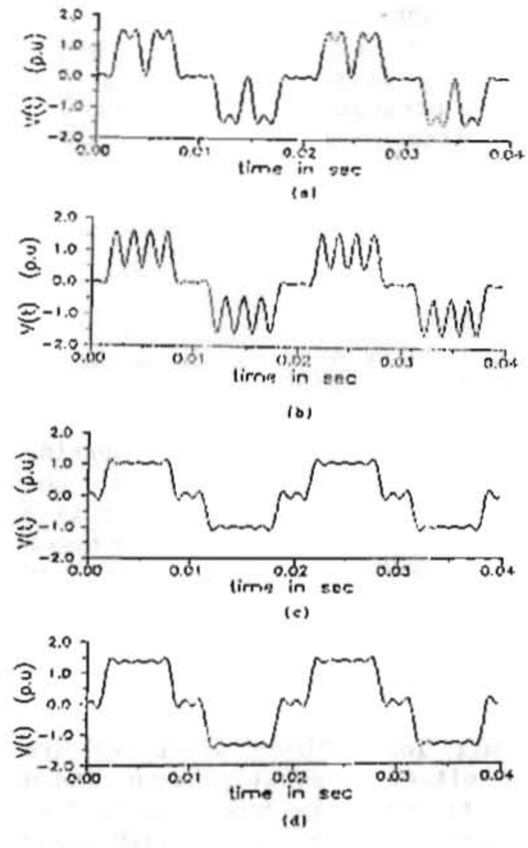

Fig. (2) Comparison of Voltage waveform.
(a) rpw =0.75, 2m=2 (b) rpw=0.75, 2m=4
(c) rpw =0.75, 2m=6 (d) rpw=1.0 , 2m=2

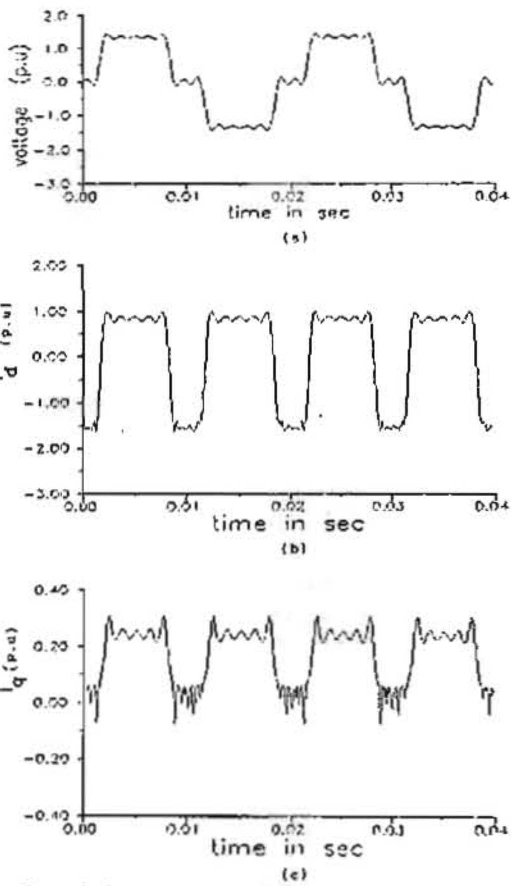

Fig. (3) Voltage and current waveforms at no-load, rpw=1.0, 2m=2
(a) V (b) i_d (c) i_q

When the PMSM is fed from pulse - width modulation inverter the voltage applied to the motor is the summation of harmonic contents obtained from Fourier series as follows :

$$V(\omega t) = \sum_{n=1}^{\infty} \left[a_n \sin (nwt) + b_n \cos (n\omega t) \right] \tag{18}$$

Figure (2) shows the comparison of output voltage waveforms for rpw =1.0 and 0.75 at different values of number of pulses per half-cycle. It is noted that, for the lower values of the number of pulses per half-cycle and rpw = 0.75, the high frequency ripples occur in the voltage waveform, while for a six-pulses per half cycle, the voltage waveform approach the same output voltage as that with rpw =1.0 mode. The high frequency ripples in the output voltage will cause non-uniform magnetic flux and higher motor heating and higher level of noise and vibration.

5. STEADY-STATE PERFORMANCE

This section deals with the analysis of the steady state performance of PMSM with and without voltage-frequency control using pulse-width modulation. The PMSM used in this investigation is a 4 hp, 2-pole, 3-phase, 230 volts [13]. The waveforms of the time-invarient of direct, quadrature current and magnetic torque are presented. The performance is carried out for two operating conditions, no-load and full-load and with rpw= 1.0 and 0.75, which have proved to be the most efficient and suitable values.

5.1 Direct and Quadrature Currents

The direct and quadrature (d,q) current waveforms when the motor is operated with inverter at no-load and rpw=1.0 are shown in Fig.(3). It can be noted that the spikes in the direct and quadrature current waveforms are produced due to spikes of the voltage waveforms. Comparing Fig.(3-a,b) with those without inverter indicate the increase in the harmonic content which will affect considerably the motor starting performance. Figure(4) shows the d and q current waveforms when the motor is operated with inverter at no-load and rpw=0.75 , 2m=2. This case simulates the worst mode of operation, because the harmonic content is large if compared with Fig.(3) (rpw=1.0, 2m=2). The d & q current waveforms contain two pulses per half-cycle according to the input voltage.

Fig.(5) shows the d & q current waveforms when the motor is operated without inverter and is subjected to full load. These waveforms are sinusoidal with distorted spikes when the voltage is equal to zero. The d & q current waveforms when the motor is connected to inverter, rpw=1.0,at full load torque are shown in Fig.(6). Comparing these results with those of Fig.(5) demonstrate the effect of the inclusion of inverter. The shape of the current has a flat top with ripples instead of nearly sinusoidally positive half-wave and the oscillation in the off-period has been improved if compared with the case of without inverter. When the rpw=0.75 , 2m=2 and the motor is subjected to full load torque, the d & q current waveforms are shown in Fig.(7). The influence of inverter becomes more dramatic in terms of motor current distortions. This case simulates the worst mode of operation, because the current waveforms are highly distorted

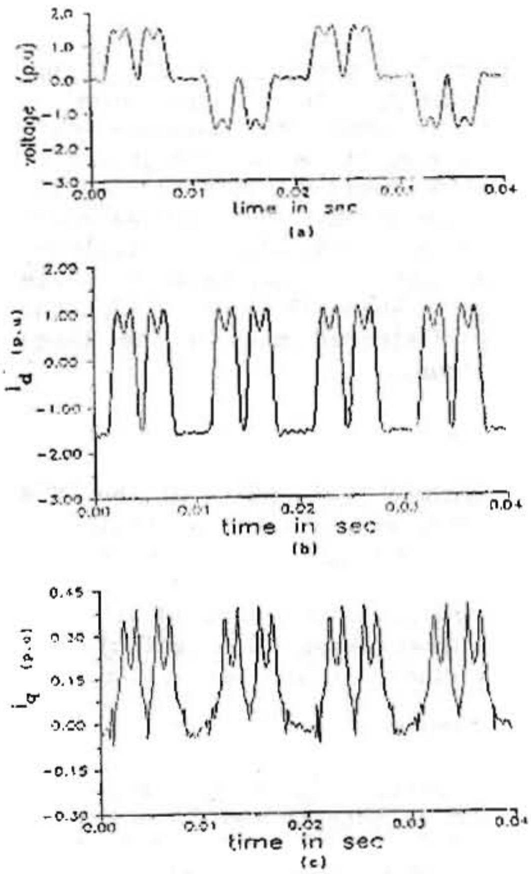

Fig. (4) Voltage and current waveforms at no-load, rpw =0.75 , 2m=2
(a) V (b) i_d (c) i_q

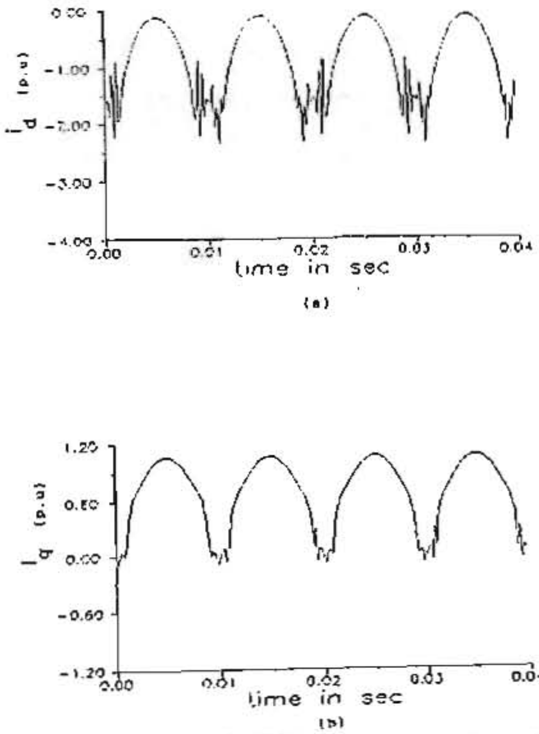

Fig. (5) Current waveforms without inverter at full load (a) i_d (b) i_q

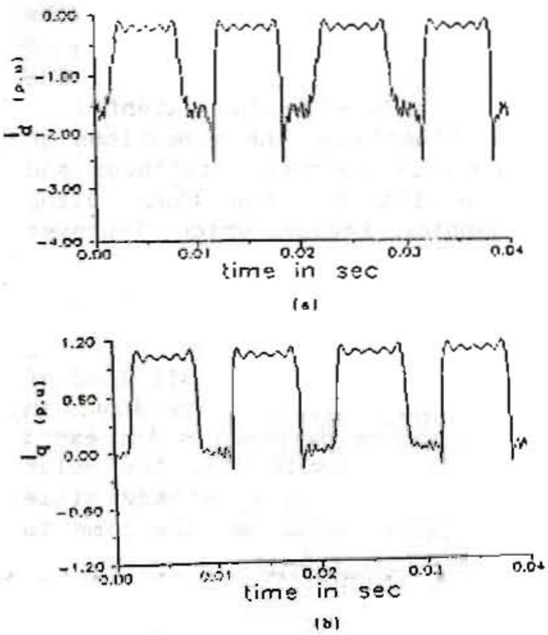

Fig. (6) Current waveform at full load rpw = 1.0 (a) i_d (b) i_q , 2m=2

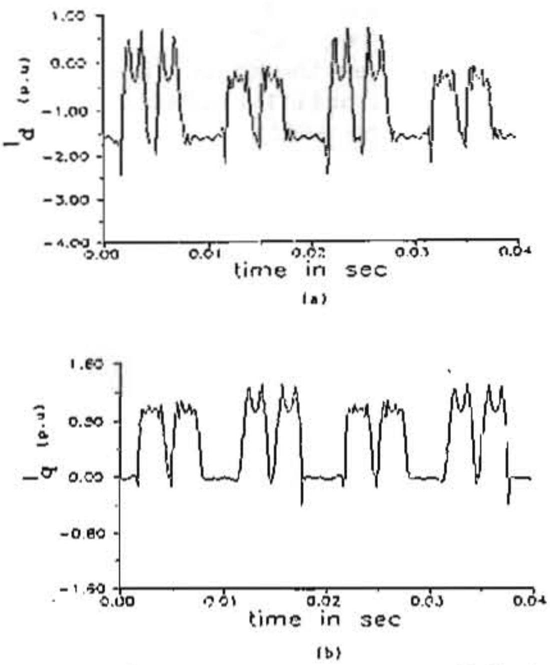

Fig. (7) Current waveforms at full load rpw = 0.75 (a) i_d (b) i_q 2m=2

5.2 Torque

This section is concerned with the simulation and analysis of the steady state motor torque performance, and to show how this performance is affected by using the speed control via the converter-inverter set. Fig. (8) shows the motor electromagnetic torque waveform when subjected to full load torque and operated without and with inverter (rpw=1.0 & 0.75). It is shown that, when the motor operates without inverter, the waveform approximately is a square-wave with off-period containing oscillations. For rpw=1.0, the electromagnetic torque has the same square-wave, while the off oscillating period is increased. When the rpw=0.75, the electromagnetic torque waveform is highly distorted due to the large values of harmonic content and pulsation torques.

6. DYNAMIC PERFORMANCE

The aim of this section is to analyze the dynamic performance of the PMSM when the motor is subjected to a small disturbance in speed or following load changes. This analysis is based on the damping and synchronizing torque technique which is described in details in section 3. A comparison of the dynamic performance when the motor is operated with and without inverter is comprehensively investigated. The effects of frequency change on the motor dynamic performance are also discussed.

6.1 Damping and Synchronizing Torque Coefficients

The damping and synchronizing torque coefficients algorithm, eqns. (12) to (15), have been used to compute the values of both K_D and K_S. A higher motor damping coefficient K_D indicates well-damped oscillations following a small load change. Moreover, higher values of the synchronizing torque coefficient K_S means fast pulling back into synchronism. The authors experience in the dynamic performance analysis of electrical machines indicates that a value of K_D = 0.03 P.u/rad/sec is adequate to provide reasonable damping of hunting oscillations. Increasing K_D above this figure reduces the rotor first swing which is a measure of an increase in the motor stability reserve. On the other hand, reducing this value increases the amplitude of the hunting oscillations to the extent that the motor may loose stability. Table (1) illustrates the comparison of torque coefficient values at different modes of operation (without and with inverter, rpw=1.0 & 0.75 , 2m=2). The results show that, using inverter with rpw=1.0, increases the damping factor which improves stability, while with rpw=0.75 the vice versa.

6.2 Load Change

The effects of load changes on the load angle responses at full load of the motor without and with inverter, rpw =1.0 & 0.75, 2m=2 are shown in Fig. (9). It is clear that, for rpw =1.0, the motor damping is increased as mentioned in the above section and shown in Table (1), the motor first swing is reduced and fast response is reached to steady state value. For rpw =0.75, the motor load angle first swing and the time to reach steady state values and its final value is increased.

6.3 Motor Currents and Torque

The direct and quatrature current responses as a function of time at full load with step load change, with and without inverter are shown in Fig. (10). It is shown that, for rpw =1.0, a fast response is reached to

Table (1) Comparison of torque coefficients at 50HZ

	mode of operation	without	rpw=1.0 2m=2	rpw=0.75 2m=2
K_e	no load	0.025791	0.030761	0.022370
	full load	0.036016	0.041900	0.031868
K_s	no load	0.012341	0.012030	0.012361
	full load	0.016600	0.017308	0.015923

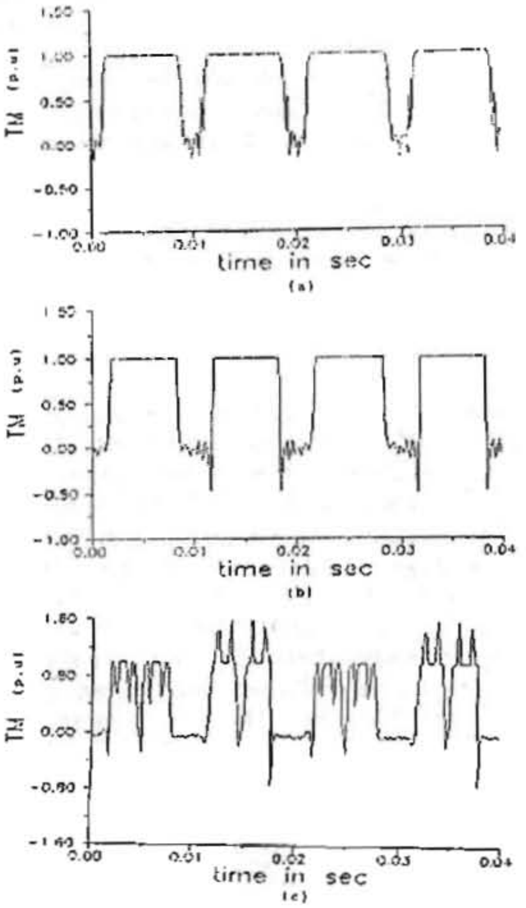

Fig. (8) Magnetic torque waveform at full load
(a) without inverter (b) rpw=1.0 , 2m=2
(c) rpw=0.75

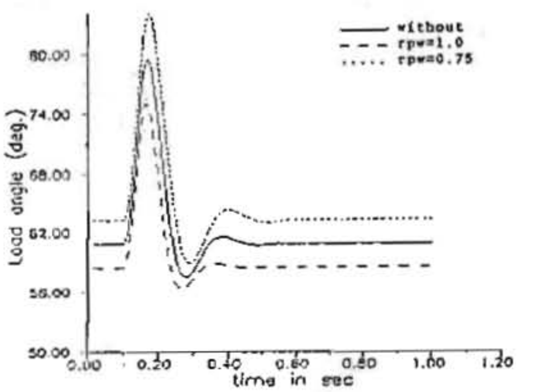

Fig. (9) Comparison of load angle response at full load , 2m=2

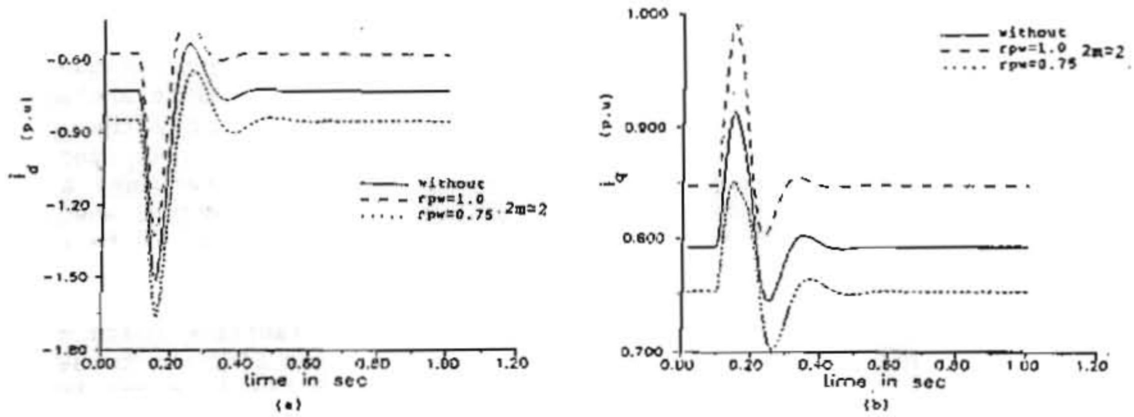

Fig. (10) Comparison of current response at full load (a) i_d (b) i_q

steady state value, and the second undershoot is reduced in amplitude and duration. The second undershoot of rpw = 0.75 mode of operation is increased in amplitude if compared with or without inverter. The steady state values of direct and quadrature currents are reached in less than one second and are different in amplitude. The electromagnetic torque-time waveform at full load with and without inverter are shown in Fig. (11). When the motor is subjected to load change with inverter (rpw = 1.0), it is noted that, the duration of swings is decreased, while for rpw = 0.75 the duration of swings is slightly increased if compared with the case of without inverter. The steady state value of these operation modes are reached in less than one second.

6.4 Frequency Variation Effects

The speed of the PMSM is uniquely related to the command frequency. The frequency to voltage proportionally defines the maximum torque available from the motor. When the d.c link voltage reaches its maximum value, the motor enters into constant power region, i.e further increase of frequency over optimum range decreases the available torque due to reduction of air gap flux. The effects of frequency change on direct and quadrature current-time response at full load torque at 25 Hz and 75 Hz are shown in figs. (12) and(13) respectively. The oscillatory condition is produced when the motor operates at 25 Hz, and the steady state is not reached. When the motor operates at 75 Hz the swings are reduced and gives fast response if compared withthe response of fig. (10) when the motor operates at 50 Hz. Fig. (14) shows the effects of frequency on the electrical torque-time response when the motor is subjected to full load torque at 25 & 100 Hz. The oscillations occur due to the 25 Hz motor operation and the setting time has not been reached. This case of operation simulates the instability due to the frequency reduction. When the motor operates at 75 Hz, the torque-time response is improved if compared with fig. (11), at 50 Hz, due to the reduction of the duration of swings and elimination of the second overshoot. It is also found, from fig. (14c), that within the range of frequency from 35 to 100 Hz, the motor operates without loosing stability and without any distortion in the time response during dynamic performance. It is concluded that if the PMSM is operated at a frequency range under 35 Hz the load torque must be reduced to keep the motor operation in the stable region.

The effects of frequency on speed deviation and torque coefficients when the motor operates at no-load and full-load torque step change are shown in table (2). It is noted that, when decreasing the frequency the speed deviation is increased. For 25 Hz mode of operation the motor operates out of synchronism. The lower value of optimum range of frequency at 35 Hz gives a stable operating mode and a suitable speed deviation. When the frequency is increased over 50 Hz the speed deviation is equal to zero, and the motor operates smoothly. It is also noted, from table (2), that the torque coefficients K_D, K_S are proportional to frequency. As explained before the increase in the damping factor improve stability which means that using inverter for the optimum range of frequency will not cause any stability problem while using inverter under this range is possible but needs a reduction of the motor load. When the motor operates at no-load the change of the torque coefficients with frequency is approximately linear relation.

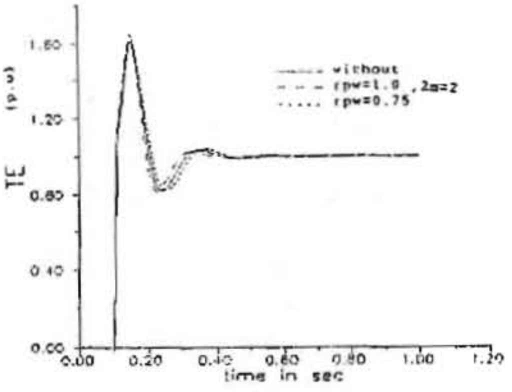

Fig. (11) Electrical torque resopnse at full load

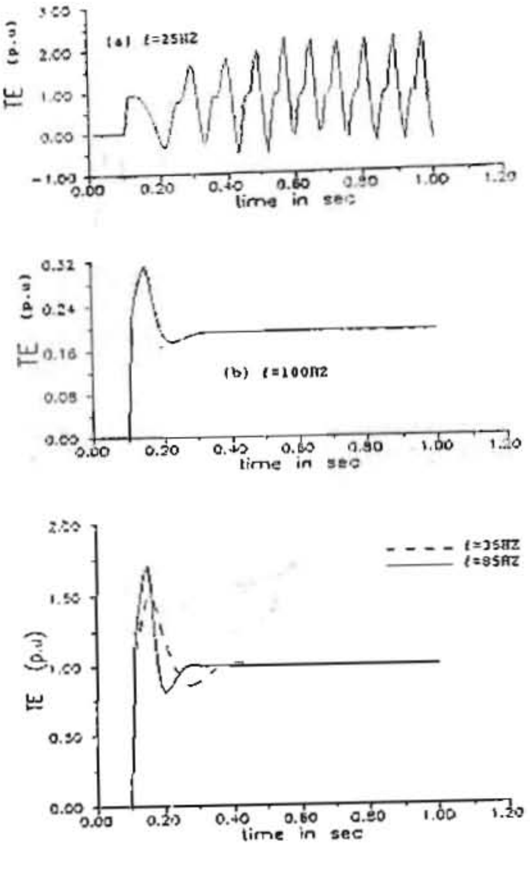

Fig. (14) Effect of frequency on the electrical torque-time response at full load

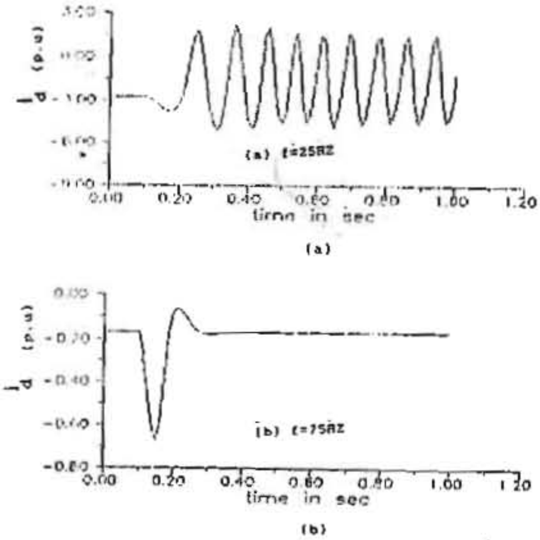

Fig. (12) Effect of frequency on i_d at full load (a) f=25 Hz , (b) f=75 Hz

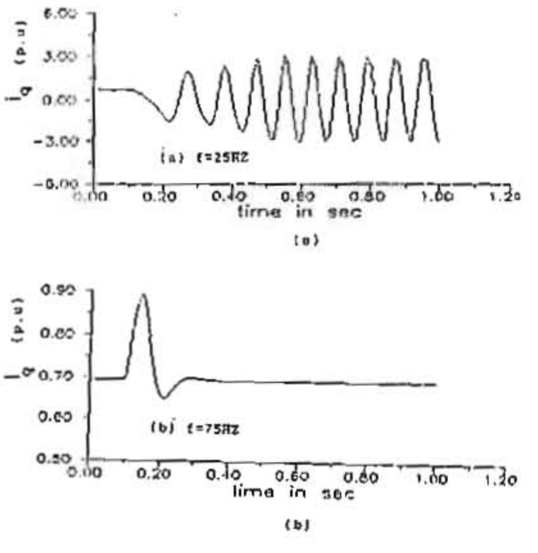

Fig. (13) Effect of frequency on I_q at full load (a) f= 25 Hz, (b) f= 75 Hz

Table (2) Torque coefficients and speed deviation at different values of frequency

Frequency	K_D (p.u/rad/sec)		K_S (p.u/rad)		Speed deviation (rad/sec)	
	no load	full load	no load	full load	no load	full load
25	0.016018	0.004989	0.006277	0.000018	0.009492	76.824304
35	0.021924	0.031258	0.00855	0.013101	0.002394	-0.000196
50	0.030761	0.041900	0.012030	0.017308	0.000003	-0.000001
75	0.045434	0.056486	0.017910	0.023054	0.000000	0.000000
85	0.051286	0.062026	0.020287	0.025220	0.000000	0.000000
100	0.060054	0.060604	0.023884	0.023574	0.000000	0.000000

7. CONCLUSIONS

This paper has presented the steady state and dynamic performance analysis of (PWM)inverter-fed PMSM. A computer-aided simulation model for predicting the performance was formulated using damping and synchronizing torques, which facilitates considerably the ability to compare the simulation without and with (PWM)inverter used. A comprehensive set of results for the digital simulation of the performance of a PMSM with and without inverter used has been obtained. Main conclusions drawn from this analysis are as follows :

(i) Using (PWM)inverter has no adverse effect on the overall motor steady state performance.

(ii) An (PWM)inverter with rpw =1.0 increases the motor damping more than the case without inverter, this means improving the motor dynamic performance if the motor is subjected to load change or small disturbance.

(iii) Using (PWM)inverter within the defined optimum operating frequency range (35-100 Hz) has no effect on the motor dynamic stability when the motor is fully loaded. The frequency operating range may increased specially the lower range if the motor is operating under light load.

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List of Symbols

- E : Voltage produced from the PM
- I, i : Current, P. u
- K_D : Damping torque coefficient
- p : Differential operator
- rpw : Relative pulse-width
- t : Time
- X : Reactance, P.u
- ω : Angular velocity, rad/sec.
- ψ : Flux linkage
- α : Triggering angle
- f : Supply frequency, Hz
- J : Motor inertia
- K_s : Synchron. torque coefficient
- R : Resistance, P.u
- s : Slip
- T : Torque
- V : Voltage, P.u
- δ : Load angle
- θ : Rotor displacement

Subscripts

- d : Direct axis
- q : Quadrature axis
- D : Direct axis damper circuit
- Q : Quadrature axis damper circuit
- n : Harmonic order
- M : Motor
- L : Load